Lecture 3

Tuesday, September 6, 2016 8:52 AM

Office Hours (Starting Next Week)

- Wed, Fri – 11:00 am – 12 pm
  Hamilton Hall 407

Inverse func.

f is 1-1 i.e whenever $x_1 \neq x_2$, $f(x_1) \neq f(x_2)$.

- $f^{-1}(y) = x$ if and only if $f(x) = y$

Cancellation equations :

$f^{-1}(f(x)) = x$ for all x in A
$f(f^{-1}(x)) = x$ " " x in B

How to find inverse functions ?

Ex Find the inverse of $f(x) = 2x+4$.

Step 1 Write $y = f(x)$

- $y = \underbrace{2x+4}$

Step 2 Solve the equation for x.

$y = 2x+4 \Rightarrow y-4 = 2x$
$\Rightarrow \frac{y-4}{2} = x$

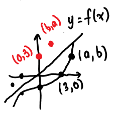
Step 3 To express f^{-1} as a function of x, switch x & y.

$y = \frac{x-4}{2} \Rightarrow f^{-1}(x) = \frac{x-4}{2}$

DIY Check $f(f^{-1}(x)) = x$
$f^{-1}(f(x)) = x$

Graphs of inverse function

DIY



$f(3) = 0$
$\Updownarrow$
$f^{-1}(0) = 3$
$\Updownarrow$
(0,3) lies on the graph of f^{-1}

We can get from (a,b) to (b,a) by reflecting the point about the line $y = x$.
Therefore, the graph of $f^{-1}(x)$ is obtained by reflecting the graph of f about the line $y = x$.

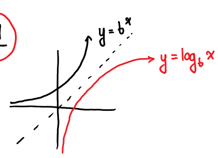
Logarithmic function

- $b > 0$, $b \neq 1$. $f(x) = \underline{b^x}$ is increasing or decreasing, hence 1-1. (CIY)
  exponential func.
- Hence it has an inverse fn, which we denote by $\log_b x$
- $\log_b x = y \Leftrightarrow b^y = x$

Cancellation equations

- $\log_b(b^x) = x$ for all $x \in \mathbb{R}$ ← Real numbers $(-\infty, \infty)$
  belongs to
- $b^{\log_b x} = x$ for all $x \in (0, \infty)$

$b > 1$



Properties of logarithms

1) $\log_b(xy) = \log_b x + \log_b y$
2) $\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$
3) $\log_b(x^r) = r\log_b x$

- $a^x . a^y = a^{x+y}$
- $\frac{a^x}{a^y} = a^{x-y}$
- DIY Show 1, 2

Ch 1.5

Natural Log

$\ln x = \log_e x$, $e \approx 2.7182....$

$\ln y = x \Leftrightarrow \log_e y = x \Leftrightarrow e^x = y$

Cancellation Eqs

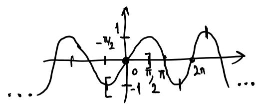
$\ln e^x = x$ for all $x \in \mathbb{R}$

$e^{\ln x} = x$ for all $x \in (0, \infty)$

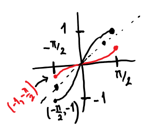
Inverse Trigonometric functions

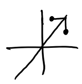
$f(x) = \sin x$

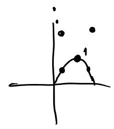
Problem: $\sin x$ is not 1-1.



The problem is overcome by restricting domain so that the function becomes 1-1.

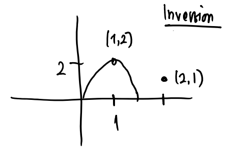






$\left(x, \frac{1}{y}\right)$

Inversion



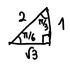
$y = \sin x$, $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$ is 1-1

and hence we can define it's inverse func. which we denote by $\sin^{-1}$ or arcsin.

- $\sin^{-1}(x) = y \Leftrightarrow \sin y = x$ and $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

$f(-x) = -f(x)$

Ex $\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$



$\Updownarrow$

$\sin\theta = \frac{1}{2}$ and $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

$\Updownarrow$

$\theta = \frac{\pi}{6}$

$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$ $\Rightarrow \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$

$\Rightarrow \sin\left(\frac{-\pi}{6}\right) = -\sin\left(\frac{\pi}{6}\right) = -\frac{1}{2}$

$-\frac{\pi}{2} \le x \le \frac{\pi}{2}$

$\sin\left(-\frac{\pi}{6}\right) = -\sin\left(\frac{\pi}{6}\right)$

$= -\frac{1}{2}$

$\sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$

$\sin x$ is an odd function.

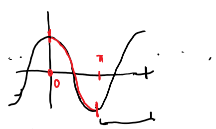
- $\sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$

- $\sin^{-1}$ has domain $[-1, 1]$, range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\sin x = -\frac{1}{2} \Rightarrow \sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$

Inverse of cosine is handled similarly.

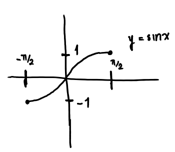
The restricted cosine function $f(x) = \cos x$, $0 \le x \le \pi$.



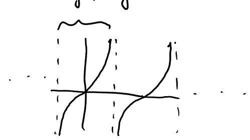
- $\cos^{-1}$ or arccos

i.e $\cos^{-1}(y) = x \Leftrightarrow \cos x = y$, $0 \le x \le \pi$

$\cos^{-1}$ has domain $[-1, 1]$, range $[0, \pi]$



- Similarly for $y = \tan x$



$y = \tan x$, $-\frac{\pi}{2} < x < \frac{\pi}{2}$

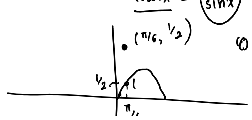
- $\tan^{-1}(x)$, arctan

$\arctan(y) = x \Leftrightarrow \tan x = y$, $-\frac{\pi}{2} < x < \frac{\pi}{2}$

Domain of arctan $\mathbb{R}$.

Range $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

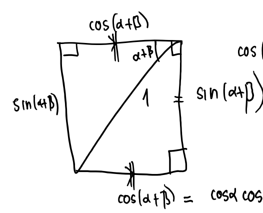
$\operatorname{cosec} x = \frac{1}{\sin x}$ → reciprocation.



$\operatorname{cosec}\frac{\pi}{6} = \frac{1}{\sin\frac{\pi}{6}} = \frac{1}{\frac{1}{2}} = 2$

Appendix D

Ch 1.5



$\cos(\alpha+\beta) = \frac{x}{1}$

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

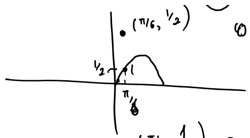
$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$$\frac{\frac{\sin x\cos y + \cos x \cdot \sin y}{\cos x \cdot \cos y}}{\frac{\cos x\cos y - \sin x\sin y}{\cos x \cdot \cos y}} = \frac{\tan x + \tan y}{1 - \tan x\tan y}$$

$\arctan(y) = x \iff \tan x = y, \ -\frac{\pi}{2} \quad \frac{\pi}{2}$

Domain of arctan $\mathbb{R}$.

Range $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.



$\csc\frac{\pi}{6} = \frac{1}{\sin\frac{\pi}{6}} = \frac{1}{\frac{1}{2}} = 2$

$\left(\frac{\pi}{6}, \frac{1}{2}\right) \rightsquigarrow \left(\frac{1}{2}, \frac{\pi}{6}\right)$

Cancellation $\quad f(x) = y \iff f^{-1}(y) = x$

$f^{-1}(f(x)) = x$ ✓

$f^{-1}(y) = x$

$y = f^{-1}(x)$

$\Downarrow$

$f(y) = x$

$f(\underbrace{f^{-1}(x)}_{y}) = x$

$= f(y) = x$

	D	R
$\cos x$,	$0 \le x \le \pi$	$[-1, 1]$
$\cos^{-1}$	$[-1, 1]$	$[0, \pi]$

$8 - \sqrt{x-9} \geqslant 0$

$x \geqslant 9$

$8 \geqslant \sqrt{x-9}$

$\underbrace{\ln(8 - \sqrt{x-9})}_{\substack{\vee \\ 0}}$

$8 - \sqrt{x-9} > 0$

$x \geqslant 9$

$\sqrt{x-9} < 8$

$x - 9 < 64$

9

$x < 73$

$f(x) = 4 + x + \ln(x-2)$

$y = 4 + x + \ln(x-2)$

$x = 4 + y + \ln(y-2)$

$x - 4 = y + \ln(y-2)$

- $e^{x-4} = e^y \cdot (y-2)$
- $e^{x-y} = e^y \cdot e^{\ln(y-2)}$.
- $4 + x + \ln(x-5) = y$

$e^{4+x} + (x-5) = e^y$

$f^{-1}(7) = ?$

$f(y) = 7$

$4 + x + \ln(x-5) = 7$

$x + \ln(x-5) = 3$

$e^x + x - 5 = e^3$

$e^x + x = e^3 + 5$

$4 + x + \ln(x-5) = 7$

$x + \ln(x-5) = 3$

$e^x \cdot (x-5) = e^3$

$e^{x-3} \cdot (x-5) = 1$

$e^{x-3} = \frac{1}{x-5}$